



Douglas College

Douglas College Learning Centre

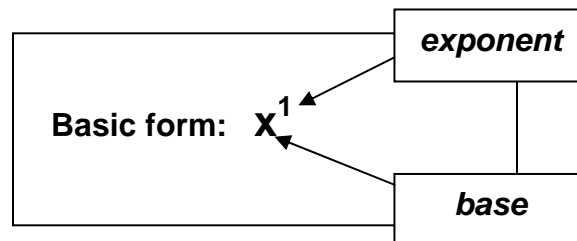
EXPONENTS

An Introduction

To use this worksheet you should first be comfortable with basic multiplication, division and fractions. If you aren't, please review them with your tutor.

Exponents are common in math because they can make writing equations much faster and easier. At first, they may seem difficult to use and understand, but with practice you'll be able to calculate them without much effort at all.

An *exponent* refers to a small number that is written above another number on the right-hand side. Sometimes, the exponent is written above a *variable*, a letter that represents a number. The number or variable that is below the exponent is called the *base*.



An exponent represents repeated multiplications of its base. To calculate an exponent, you multiply the base by the base for as many times as the exponent indicates. Here are some steps you can follow:

Step 1: Write the base out as many times as the exponent indicates and write multiplication signs between each.

Step 2: Multiply all the bases together one at a time.

Now let's try an example by calculating 2^4 .

Step 1) The exponent is 4, so we know to multiply 2, the base, by itself 4 times.

Step 2) $2^4 = 2 \times 2 \times 2 \times 2$
Now we multiply the bases one at a time.

$$\begin{array}{c} 2 \times 2 \times 2 \times 2 \\ \underbrace{\hspace{1.5cm}} \\ 4 \times 2 \times 2 \\ \underbrace{\hspace{1.5cm}} \\ 8 \times 2 \\ \underbrace{\hspace{1.5cm}} \\ 16 \end{array}$$

So now we know that $2^4 = 16$.

Exercise 1: Basic Exponents

Write the equation as a base with an exponent.

1) $2 \times 2 \times 2 \times 2 \times 2$

2) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$

3) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

4) 5×5

Two common exponents that people sometimes make mistakes with are 1 and 0. A base with an exponent of 1 is easier to calculate than you might think. It simply equals the base:

$$\begin{array}{l} 2^1 = 2 \\ a^1 = a \end{array}$$

A base with an exponent of 0 is also easy to calculate. It simply equals 1:

$$\begin{array}{l} 2^0 = 1 \\ a^0 = 1 \end{array}$$

Exercise 2: Basic Exponents

Calculate the exponent.

5) 5^4

6) 6^1

7) 2^7

8) 4^0

Let's use the same steps we learned earlier to calculate a trickier example, $3^3 \times 3^2$.

Step 1) First, let's write out the bases of both the exponents. We'll use brackets to make it easier to follow.

$$3^3 \times 3^2 = (3^3) \times (3^2) = (3 \times 3 \times 3) \times (3 \times 3) = 3 \times 3 \times 3 \times 3 \times 3$$

Step 2) Next we multiply the bases together.

$$\begin{array}{r} 3 \times 3 \times 3 \times 3 \times 3 \\ \underbrace{\hspace{1.5cm}} \\ 9 \times 3 \times 3 \times 3 \\ \underbrace{\hspace{1.5cm}} \\ 27 \times 3 \times 3 \\ \underbrace{\hspace{1.5cm}} \\ 81 \times 3 \\ \underbrace{\hspace{1.5cm}} \\ 243 \end{array}$$

In the first step of the example above, we found that $3^3 \times 3^2 = 3 \times 3 \times 3 \times 3 \times 3$. Isn't this the same as what 3^5 would equal?

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$$

$$3^3 \times 3^2 = 3^5$$

Notice that when added together the exponents above, 3 and 2, equal the other exponent, 5. This example shows an important rule:

Rule 1:

When multiplying exponents with the same base, add the exponents together:

$$a^n \times a^m = a^{n+m}$$

Just as you add exponents when you multiply two equivalent bases, you subtract exponents when you divide two equivalent bases. For example:

$$4^5 \div 4^2 = \frac{4^5}{4^2} = \frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4} = 4 \times 4 \times 4 = 4^3$$

$$4^5 \div 4^2 = 4^{5-2} = 4^3$$

Rule 2:

When dividing exponents with the same base, subtract the exponents:

$$a^n \div a^m = a^{n-m}$$

Note that adding or subtracting exponents doesn't work when the bases that are being multiplied or divided are different. For example:

$$4^5 \div 3^2 \neq 4^{5-2} \text{ or } 3^{5-2}$$

$$a^5 \times b^2 \neq a^{5+2} \text{ or } b^{5+2}$$

Exercise 3: Multiplying and dividing

Evaluate each exponent. Express your answer as a base with an exponent

9) $6^{18} \div 6^{16}$

10) $2^3 \times 3^4$

11) $a^3 \times a^2$

12) $y^{40} \div y^{27}$

Sometimes you may run into a negative exponent. Look at the following three equations and see if you can spot what it means.

$$a^{-3} = \frac{1}{a^3}$$

$$\frac{1}{z^{-4}} = z^4$$

When you encounter a base with a negative exponent, take the base's *reciprocal* and make its exponent positive. To take a reciprocal of a number, you make it a fraction and then reverse its *numerator*, the number on the top of the fraction, and its *denominator*, the number on the bottom of the fraction. Therefore,

$$a^{-3} = \frac{a^{-3}}{1} = \frac{1}{a^3}$$

Rule 3:

To calculate a negative exponent, take the base's *reciprocal* and make its exponent positive:

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \frac{1}{x^{-n}} = x^n$$

Exercise 4: Negative Exponents

Evaluate each exponent. Express your answer as a base with an exponent

13) $z^{-3} \times z^5 \div z^0$ 14) $y^2 z^{-3} \times y^4 z^3$ 15) $\frac{2}{a^{-3}} \times a^2$ 16) $y^{-40} \div y^{-60}$

Answers*Exercise 1*

1) 2^5

2) 9^{11}

3) 3^8

4) 5^2

Exercise 2

5) 625

6) 6

7) 128

8) 1

Exercise 3

9) $6^2 = 36$

10) 648

11) a^5

12) y^{13}

Exercise 4

13) z^2

14) y^6

15) $2a^5$

16) y^{20}